Analysis of Single-Phase SPWM Inverter

A THESIS SUBMITTED IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
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BACHELOR OF TECHNOLOGY
IN
ELECTRICAL ENGINEERING

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NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA

CERTIFICATE

This is to certify that the thesis entitled **"Analysis of Single Phase SPWM Inverter"** submitted by Mr. Bijoyprakash Majhi in partial fulfilment of the requirements for the award of Bachelor of Technology Degree in Electrical Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any degree or diploma.

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ABSTRACT

This project deals with studying the basic theory of a Sinusoidal Pulse Width Modulated Inverter (SPWM), its Simulink modelling, estimating various designing parameters and various instabilities. The project will be commenced by a basic understanding of the circuitry of the SPWM Inverter, the components used in its design and the reason for choosing such components in this circuitry. After this, it will be attempted to simulate a model circuit on any simulating software e.g. MATLAB and analyse the output waveforms for various values of the elements used in the circuit and hence study the system response and instabilities.

CHAPTER I

Introduction

CHAPTER I: INTRODUCTION

1.1 Inverters

An inverter is basically a device that converts electrical energy of DC form into that of AC. The purpose of DC-AC inverter is to take DC power from a battery source and converts it to AC. For example the household inverter receives DC supply from 12V or 24V battery and then inverter converts it to 240V AC with a desirable frequency of 50Hz or 60Hz. These DC-AC inverters have been widely used for industrial applications such as uninterruptible power supply (UPS), AC motor drives. Recently, the inverters are also playing an important role in various renewable energy applications as these are used for grid connection of Wind Energy System or Photovoltaic System. In addition to this, the control strategies used in the inverters are also similar to those in DC-DC converters. Both current-mode control and voltage-mode control are employed in practical applications.

The DC-AC inverters usually operate on Pulse Width Modulation (PWM) technique. The PWM is a very advance and useful technique in which width of the Gate pulses are controlled by various mechanisms. PWM inverter is used to keep the output voltage of the inverter at the rated voltage (depending on the user's choice) irrespective of the output load .In a conventional inverter the output voltage changes according to the changes in the load. To nullify this effect of the changing loads, the PWM inverter correct the output voltage by changing the width of the pulses and the output AC depends on the switching frequency and pulse width which is adjusted according to the value of the load connected at the output so as to provide constant rated output. The inverters usually operate in a pulse width modulated (PWM) way and switch between different circuit topologies, which means that the inverter is a nonlinear, specifically piecewise smooth system. In addition to this, the control strategies used in the inverters are also similar to those in DC-DC converters. Both current-mode control and voltage-mode control are employed in practical applications. In the last decade, studies of complex behaviour in switching power converters have gained increasingly more attention from both the academic community and industry. Various kinds of nonlinear phenomena, such as bifurcation, chaos, border collision and coexisting attractors, have been revealed. Previous work has mainly focused on DC power supply systems including DC-DC converters and AC-DC power factor correction (PFC) converters.

CHAPTER II

Background and Literature

CHAPTER II: Background and Literature

2.1. Voltage Source Inverter and Current Source Inverter

2.1.1. Voltage Source Inverter: The type of inverter where the independently controlled ac output is a voltage waveform. The output voltage waveform is mostly remaining unaffected by the load. Due to this property, the VSI have many industrial applications such as adjustable speed drives (ASD) and also in Power system for FACTS (Flexible AC Transmission).

2.1.2. Current Source Inverter: The type of inverter where the independently controlled ac output is a current waveform. The output current waveform is mostly remaining unaffected by the load. These are widely used in medium voltage industrial applications, where high quality waveform is required.

2.2. Single Phase Half Bridge And Full Bridge VSI Inverter:

2.2.1. Single Phase Half Bridge Inverter: It consists of two semiconductor switches T1 and T2. These switches may be BJT, Thyristor, IGBT etc with a commutation circuit. D1 and D2 are called Freewheeling diode also known as the Feedback diodes as they feedback the load reactive power.

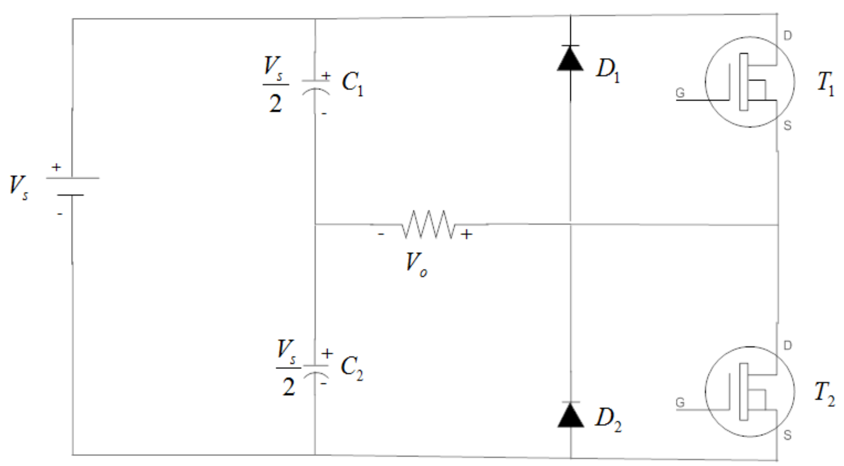


Fig1: Single Phase Half Bridge Inverter

T1	**T2**	***Vo***
ON	OFF	$+\frac{Vs}{2}$
OFF	ON	$-\frac{Vs}{2}$

Table 1: Switching States

T1 is ON during the positive half cycle of the output voltage, which makes Vout=Vo/2 and T2 is ON during the negative half cycle which makes Vout= -Vo/2. The both switches must operate alternatively otherwise there may be a chance of short circuiting. In case of resistive load, the current waveform follows the voltage waveform but not in case of reactive load. The feedback diode operates for the reactive load when the voltage and current are of opposite polarities.

2.2.2. Single Phase Full wave Bridge Inverter: It consists of two arms with a two semiconductor switches on both arms with antiparallel freewheeling diodes for discharging the reverse current. In case of resistive-inductive load, the reverse load current flow through these diodes. These diodes provide an alternate path to inductive current which continue so flow during the Turn OFF condition.

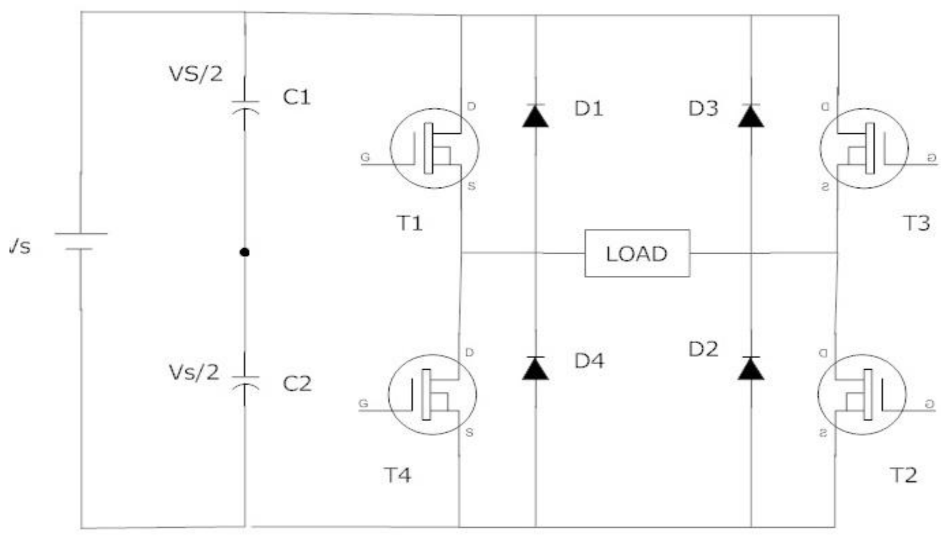


Fig2: Single Phase Full wave Bridge Inverter

T1	**T2**	**T3**	**T4**	V_A	V_B	V_{AB}
ON	OFF	OFF	ON	$\frac{Vs}{2}$	$-\frac{Vs}{2}$	V_S
OFF	ON	ON	OFF	$+\frac{Vs}{2}$	$+\frac{Vs}{2}$	-V_S
ON	OFF	ON	OFF	$\frac{Vs}{2}$	$-\frac{Vs}{2}$	0
OFF	ON	OFF	ON	$-\frac{Vs}{2}$	$+\frac{Vs}{2}$	0

Table 2: Switching States

The switches are T1, T2, T3 and T4. The switches in each branch is operated alternatively so that they are not in same mode (ON /OFF) simultaneously .In practice they are both OFF for short perod of time called blanking time ,to avoid short circuiting . The switches T1 and T2 or T3 and T4 should operate in a pair to get the output. These bridges legs are switched such that the output voltage is shifted from one to another and hence the change in polarity occurs in voltage waveform. If the shift angle is zero, the output voltage is also zero and maximal when shift angle is π.

2.3. Pulse Width Modulation (PWM): The Pulse Width Modulation (PWM) is a technique which is characterized by the generation of constant amplitude pulse by modulating the pulse duration by modulating the duty cycle. Analog PWM control requires the generation of both reference and carrier signals that are feed into the comparator and based on some logical output, the final output is generated. The reference signal is the desired signal output maybe sinusoidal or square wave, while the carrier signal is either a sawtooth or triangular wave at a frequency significantly greater than the reference.

There are various types of PWM techniques and so we get different output and the choice of the inverter depends on cost, noise and efficiency.

2.3.1. Basic PWM Techniques

There are three basic PWM techniques:

1. Single Pulse Width Modulation
2. Multiple Pulse Width Modulation
3. Sinusoidal Pulse Width Modulation

2.3.1.1. Single Pulse Width Modulation: In this modulation there is an only one output pulse per half cycle. The output is changed by varying the width of the pulses. The gating signals are generated by comparing a rectangular reference with a triangular reference. The frequency of the two signals is nearly equal.

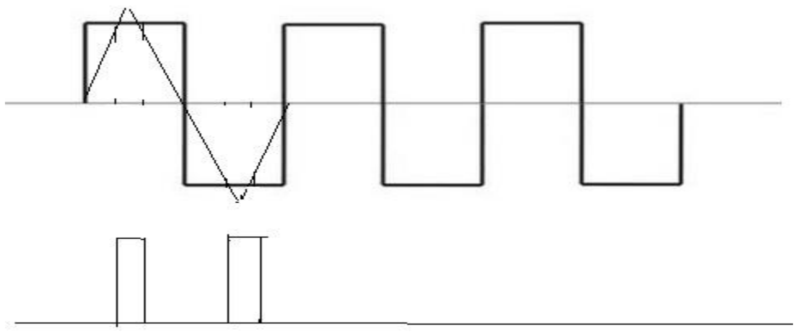


Fig3: Single Pulse Width Modulation

The rms ac output voltage

$$Vo = Vs\sqrt{\frac{2t_{ON}}{T}} = Vs\sqrt{2\delta}$$

Where

δ =duty cycle= $\frac{t_{on}}{T}$

Modulation Index (MI) = $\frac{Vr}{Vc}$

Where *Vr* = Reverence signal voltage

Vc = Carrier signal voltage

By varying the control signal amplitude V_r from 0 to *Vc* the pulse width ton can be modified from 0 secs to T/2 secs and the rms output voltage V_o from 0 to V_s.

2.3.1.2. Multiple Pulse Width Modulation: In this modulation there are multiple number of output pulse per half cycle and all pulses are of equal width. The gating signals are generated by comparing a rectangular reference with a triangular reference. The frequency of the reference signal sets the output frequency (fo) and carrier frequency (fc). The number of pulses per half cycle is determined by p: p= $\frac{f_c}{2f_o}$.

The rms ac output voltage

$$Vo = Vs\sqrt{\frac{p\delta}{\pi}}$$

Where, $\delta = dutyratio = \frac{t_{ON}}{T}$

The variation of modulation index (MI) from 0 to 1 varies the pulse from 0 to π/p and the output voltage from *0 to Vs*.

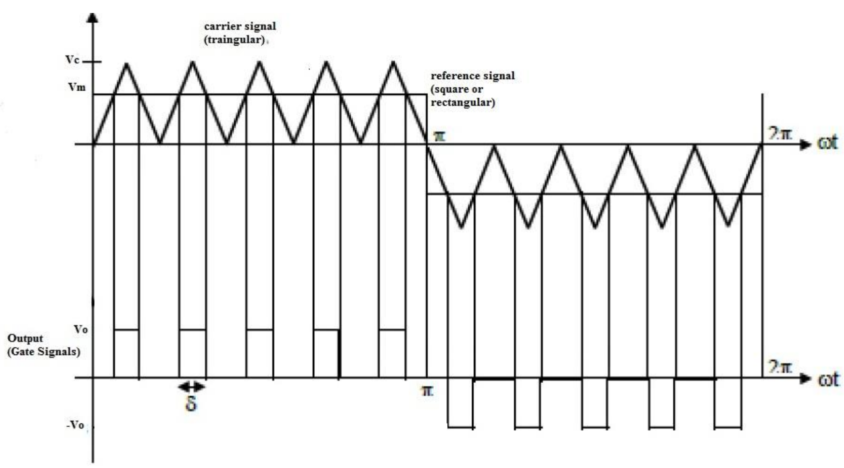


Fig4: Multiple Pulse Width Modulation

2.3.1.3. Sinusoidal Pulse Width Modulation: In this modulation technique are multiple numbers of output pulse per half cycle and pulses are of different width. The width of each pulse is varying in proportion to the amplitude of a sine wave evaluated at the centre of the same pulse. The gating signals are generated by comparing a sinusoidal reference with a high frequency triangular signal.

The rms ac output voltage

$$V_o = V_s\sqrt{\frac{p\delta}{\pi}} \rightarrow V_s\sqrt{\sum_{m=1}^{2p}\frac{\delta_m}{\pi}}$$

Where p=number of pulses and δ= pulse width

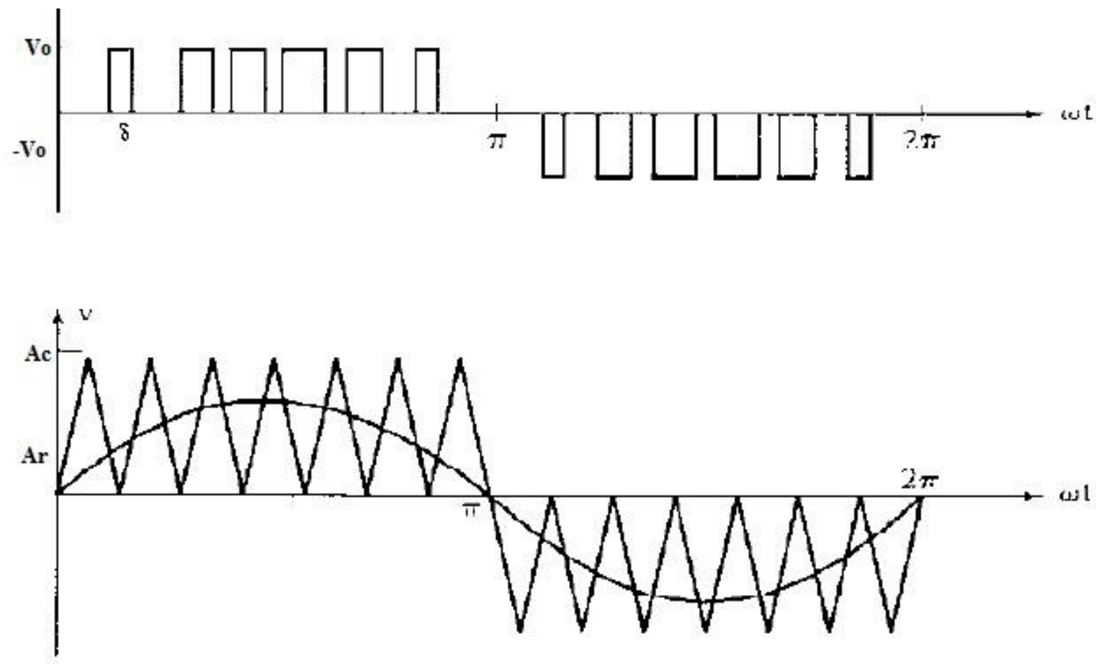


Fig5: Sinusoidal Pulse Width Modulation

2.3.1.4. Features for comparing various PWM Techniques:

- Switching Losses
- Utilization of Dc power supply that is to deliver a higher output voltage with the same DC supply.
- Linearity in voltage and current control.
- Harmonics contents in the voltage and current.

2.4. Inverter Types:

There are generally three types of inverter for general purpose

- Square Wave Inverter
- Modified Square Wave Inverter
- True Sine Wave Inverter

2.4.1. Square Wave Inverter: This is the basic type of inverter. Its output is a alternating square wave. The harmonic content in this wave is very large. This inverter is not efficient and can give serious damage to some of the electronic equipment. But due to low cost, it has some limited number of applications in household appliances.

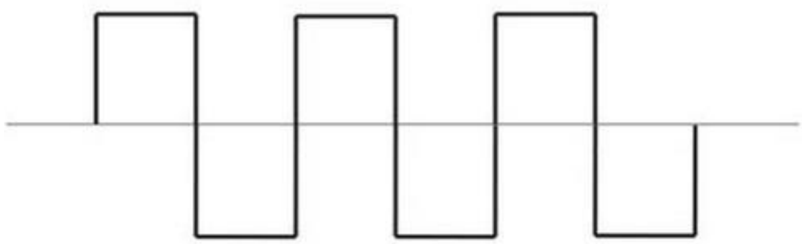


Fig6: Square Wave Inverter Output

2.4.2. Modified Square Wave Inverter: A modified sine wave inverter actually has a waveform more like a square wave, but with an extra step or so. Because the modified sine wave is noisier and rougher than a pure sine wave, clocks and timers may run faster or not work at all. A modified sine wave inverter will work fine with most equipment, although the efficiency or power will be reduced with some. But with most of the household appliances it works well.

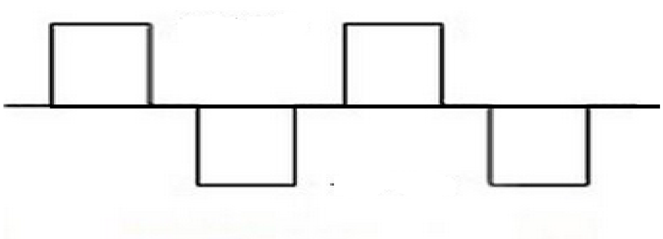


Fig7: Modified Square Wave Inverter Output

2.4.3 True Sine Wave Inverter: This type of inverter provides output voltage waveform which is very similar to the voltage waveform that is received from the Grid. The sine wave has very little harmonic distortion resulting in a very 'clean' supply and makes it ideal for running electronic systems such as computers, digital fx racks and other sensitive equipment without causing problems or noise. Things like mains battery chargers also run better on pure sine wave converters.

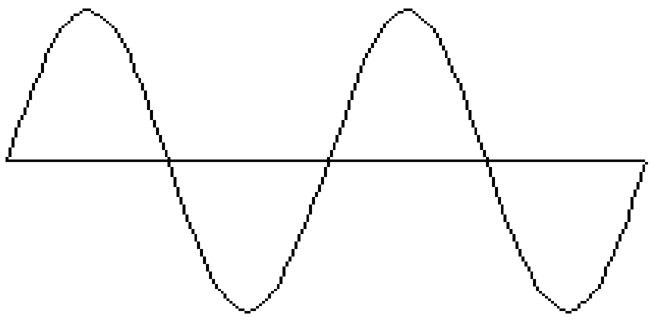


Fig8: True Sine Wave Inverter

Benefits of using True Sine Wave Inverter:

- Most of the electrical and electronic equipments are designed for the sine wave.
- Some appliances such as variable motor, refrigerator, microwave will not be able to provide rated output without sine wave.
- Electronic clocks are designed for the sine wave.
- Harmonic content is less.

2.5. Sine Wave Generation

The most common and popular technique for generating True sine Wave is Pulse Width Modulation (PWM). Sinusoidal Pulse Width Modulation is the best technique for this. This PWM technique involves generation of a digital waveform, for which the duty cycle can be modulated in such a way so that the average voltage waveform corresponds to a pure sine wave. The simplest way of producing the SPWM signal is through comparing a low power sine wave reference with a high frequency triangular wave. This SPWM signal can be used to control switches. Through an LC filter, the output of Full Wave Bridge Inverter with SPWM signal will generate a wave approximately equal to a sine wave. This technique produces a much more similar AC waveform than that of others. The primary harmonic is still present and there is relatively high amount of higher level harmonics in the signal.

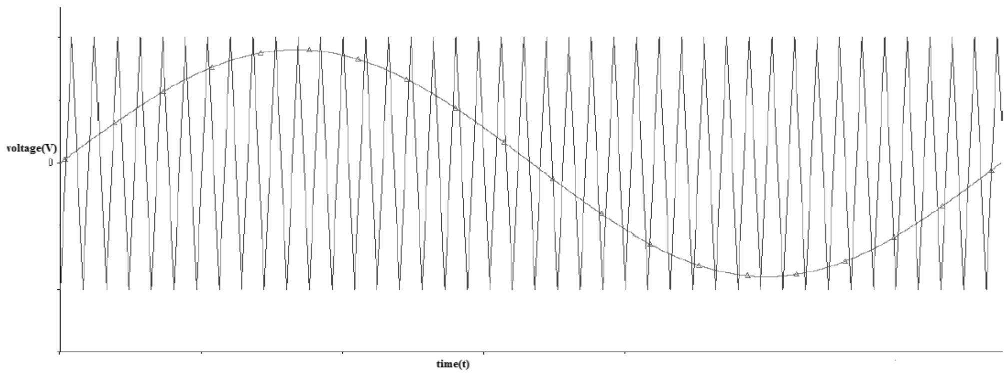


Fig9: SPWM comparison Signals

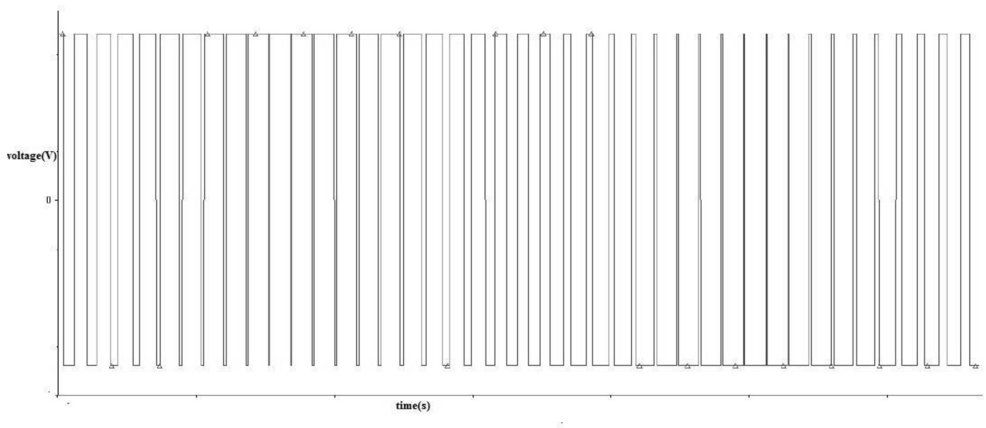


Fig10: Unfiltered SPWM output

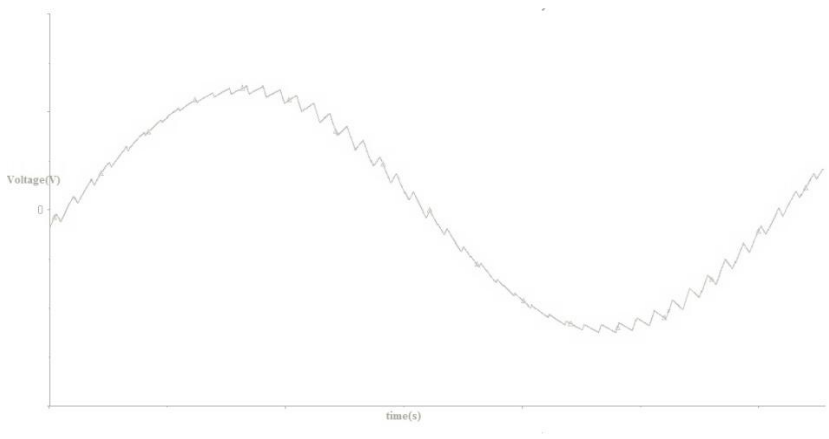


Fig11: Filtered SPWM Output

Let the modulating signal is a sinusoidal of amplitude Am, and the amplitude of the triangular carrier is Ac, the ratio m=Am/Ac is known as Modulation Index (MI). Note that controlling the MI controls the amplitude of the applied output voltage. With a sufficiently high carrier frequency . A higher carrier frequency results in large number of switching per cycle and hence increased power loss.

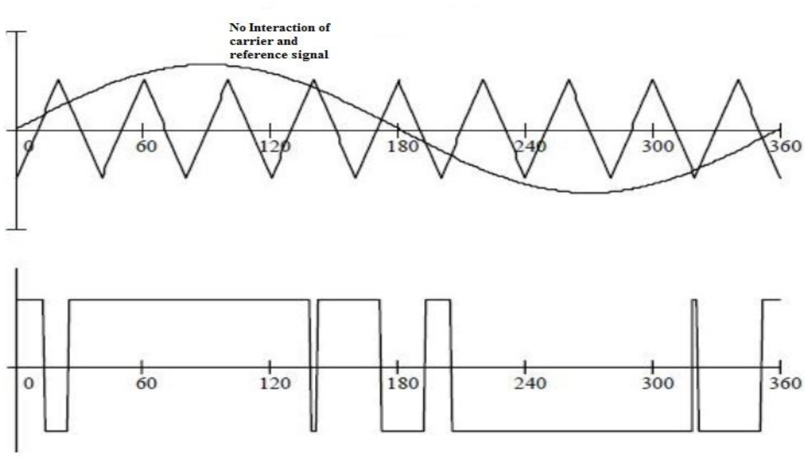


Fig12: Overmodulation

The inverting process works well for m<1 and for m>1, there are periods of the triangle wave in which there is no intersection of carrier and the signal as shown in the fig. However , a certain amount of this “overmodulation” is often allowed in the interest of obtaining a large AC voltage magnitude even though the spectral content of the voltage is poor.

2.5.1 SPWM Harmonic Elimination:

The SPWM waveform has harmonics of several orders in the phase voltage waveform , the dominant ones are the fundamental and other of order of n and n±2 where n=fc/fm. With the method of Selective Harmonic Elimination, only selected harmonics are eliminated with the smallest number of switching. For a single phase-SPWM waveform with odd and half wave symmetry and n chops per cycle as shown in fig.

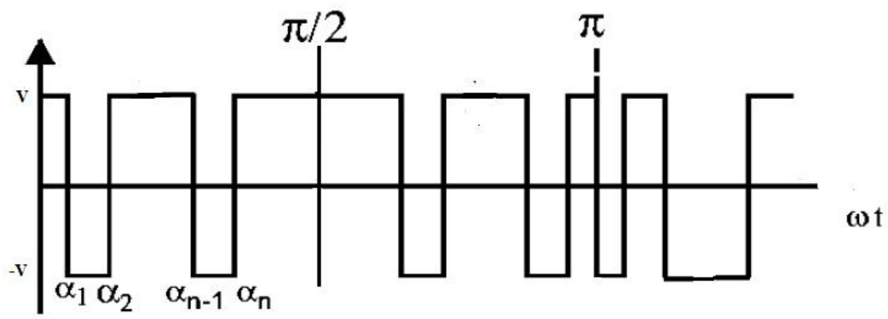


Fig13: SPWM wave with odd and half wave symmetry

The Fourier coefficients of the waveform shown in the above figure can be given by

$$h_1 = \left(4\frac{V}{\pi}\right)\left[1-2\cos\alpha_1 + 2\cos\alpha_2 - 2\cos\alpha_3 + ...2\cos\alpha_n\right] \quad (2.5.1.1)$$

$$h_3 = \left(4\frac{V}{3\pi}\right)\left[1-2\cos 3\alpha_1 + 2\cos 3\alpha_2 - 2\cos 3\alpha_3 + ...2\cos 3\alpha_n\right] \quad (2.5.1.2)$$

$$.\,h_n = \left(4\frac{V}{n\pi}\right)\left[1-2n\cos\alpha_1 + 2n\cos\alpha_2 - 2n\cos\alpha_3 + ...2n\cos\alpha_n\right] \quad (2.5.1.3)$$

Where h_n represents the magnitude of nth harmonic component and an is the primary switching angle,*n* is the no of chops.

Fourier sine and cosine coefficients are

$$a_o = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta \quad (2.5.1.4)$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} f(k\theta)\cos(k\theta)d\theta \quad (2.5.1.5)$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(k\theta)\sin(k\theta)d\theta \quad (2.5.1.6)$$

Only odd no. harmonics exist due to half cycle symmetry of waveform.

Using quarter cycle symmetry, the Fourier coefficients become

$$b_k = 4\frac{V}{\pi}\left(\int_0^{\alpha 1}\sin(k\theta)d\theta - \int_{\alpha 1}^{\alpha 2}\sin(k\theta)d\theta + \int_{\alpha 2}^{\alpha 3}\sin(k\theta)d\theta - ......\int_0^{\frac{\pi}{2}}\sin(k\theta)d\theta\right) \tag{2.5.1.7}$$

$$= 4\frac{V}{n\pi}\left[1 - 2\cos\alpha_1 + 2\cos\alpha_2 - 2\cos\alpha_3.....2\cos k\alpha_n\right] \tag{2.5.1.8}$$

SPWM is considered as the best PWM technique for the reasons mentioned below.

Advantages of SPWM:

- Low power consumption.
- High energy efficient upto 90%.
- High power handling capability.
- No temperature variation-and ageing-caused drifting or degradation in linearity.
- Easy to implement and control.
- Compatible with today's digital microprocessors

Disadvantages of SPWM:

- Attenuation of the wanted fundamental component of the waveform.
- Drastically increased switching frequencies that leads to greater stresses on associated switching devices and therefore derating of those devices.
- Generation of high-frequency harmonic components.

CHAPTER III

Methodology

CHAPTER III: Methodology

Designing a single phase inverter for household purpose or UPS (Uninterruptible Power Supply) of rating 220V or 230V, the basic things we have to design are: LC Filter ,PI controller and we have to choose an appropriate step-up Transformer.

3.1 LC Filter Design

A low pass LC filter is required at the output terminal of Full Bridge VSI to reduce harmonics generated by the pulsating modulation waveform. While designing L-C filter, the cut-off frequency is chosen such that most of the low order harmonics is eliminated. To operate as an ideal voltage source, that means no additional voltage distortion even though under the load variation or a nonlinear load, the output impedance of the inverter must be kept zero. Therefore, the capacitance value should be maximized and the inductance value should be minimized at the selected cut-off frequency of the low-pass filter.

Each value of L and C component is determined to minimize the reactive power in these components because the reactive power of L and C will decide the cost of LC filter and it is selected to minimize the cost, then **it** is common that the filter components are determined at the set of a small capacitance and a large inductance and consequently the output impedance of the inverter is so high. With these design values, the voltage waveform of the inverter output can be sinusoidal under the linear load or steady state condition because the output impedance is zero. But in case of a step change of the load or a nonlinear load, the output voltage waveform will be distorted cause by the slow system response as the output response is non-zero.

Figure 14 shows the power circuit of the single phase PWM-VSI with any linear or nonlinear load. The load current flows differently depending on the kind of loads such as linear and nonlinear load. Therefore it is difficult to represent the transfer function of inverter output voltage to load current.

The plant composed of L-C low-pass filter satisfies linear property, so it is possible to represent the system which has two inputs of inverter output voltage and load current.

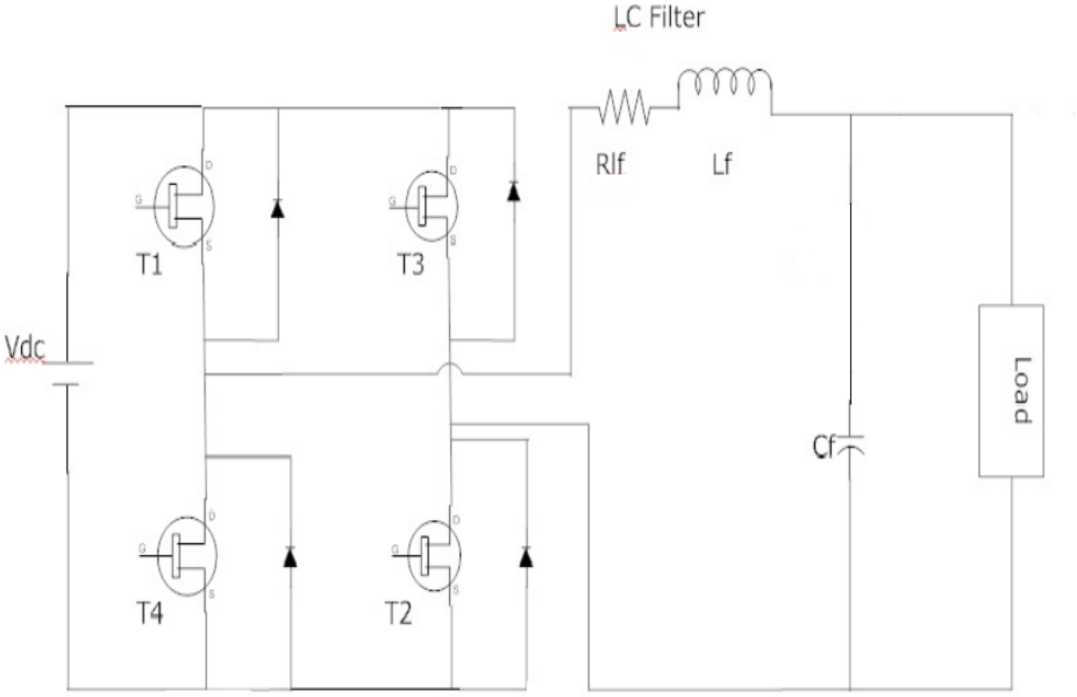


Fig14: LC Filter

Using the closed relation between the filter capacitor value and the system time constant, the capacitor value can be calculated. The effect of the load current to the voltage distortion can be calculated from the closed form. It is also possible to analyse how much the voltage waveform is distorted in the system in case of a nonlinear load.

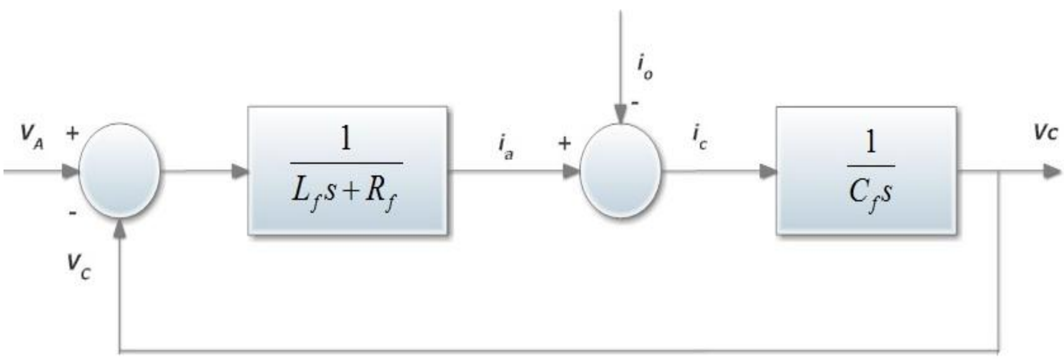


Fig15: Block Diagram of single phase PWM-VSI

Figure 15 shows the system block diagram of the single phase **PWM-VSI** and the input and output transfer function

$$V_c(s) = \frac{1}{L_f C_f s^2 + jR_f C_f} V_A(s) - \frac{L_f s + R_f}{L_f C_f s^2 + jR_f C_f s + 1} I_o(s) \tag{3.1.1}$$

The frequency transfer function can be expressed as

$$V_c(j\omega) = \frac{1}{1 - L_f C_f \omega^2 + jR_f C_f \omega} V(j\omega) - \frac{jL_f \omega + R_f}{1 - L_f C_f \omega^2 + jR_f C_f \omega} I_o(j\omega) \tag{3.1.2}$$

To determine the transfer function:

$$Va(s) - sL_f Ia(s) - R_f Ia(s) - Vc(s) = 0 \tag{3.1.3}$$

$$Va(s) - Vc(s) = Ia(s)(sL_f + R_f) \tag{3.1.4}$$

$$\frac{Va(s)}{Vc(s)} = 1 + \frac{Ia(s)(sL_f + R_f)}{Vc(s)} \tag{3.1.5}$$

$$\frac{Va(s)}{Vc(s)} = 1 + \frac{Ia(s)(sL_f + R_f)sC_f}{Ic(s)} \tag{3.1.6}$$

As $ia = ic + io$

$$Ia(s) = Ic(s) + \frac{Vc(s)}{Z_L} \tag{3.1.7}$$

$$\frac{Ia(s)}{Ic(s)} = 1 + \frac{1}{sC_f Z_L} \tag{3.1.8}$$

$$\frac{Va(s)}{Vc(s)} = 1 + (1 + \frac{1}{sC_f Z_L})(sL_f + R_f)sC_f \tag{3.1.9}$$

$$\frac{Va(s)}{Vc(s)} = \frac{s^2 L_f C_f + sL_f + R_f C_f s Z_L + R_f + Z_L}{Z_L} \tag{3.1.20}$$

$$\frac{Vc(s)}{Va(s)} = \frac{Z_L}{s2L_f C_f + sL_f + RCsZ_L + R_f + Z_L} \tag{3.1.21}$$

Now, through transfer function we can find the step response, corner or cross over frequency from bode plot and stability from root locus method.

The amplitude of the voltage harmonics at the filter output can be calculated by summing the two harmonics caused by the inverter output voltage and by the load current

The above equation can be simplified by neglecting the imaginary part in both the terms as equivalent series resistance of inductor is very small that means

$$|1 - L_f C_f \omega^2| >> |R_f C_f \omega| \qquad (3.1.22)$$

So,

$$V_c(j\omega) = \frac{1}{1 - L_f C_f \omega^2} V(j\omega) \qquad (3.1.23)$$

In the conventional output filter design method, the load current is treated **as** the disturbance so it can be neglected.

This filter design procedure can be well applied to the linear load. But in case of nonlinear load or transient load change, the output current term cannot be neglected due to the increase of load current harmonics. Therefore, for the analysis of voltage harmonics under the nonlinear load, it should be considered.

In order to be independent of the load current, the inductor value should be minimized and on the contrary maximized the capacitor value at the same cut-off frequency. Then it satisfies the zero output impedance and works **as** an ideal voltage source.

At Cut-off frequency

$$\frac{V_C(j\omega)}{V_A(j\omega)} = \frac{1}{1 - L_f C_f \omega^2} \qquad (3.1.25)$$

The filter output to input voltage harmonics must be less than 3%.

So,

$$\frac{V_C(j\omega)}{V_A(j\omega)} = 3\% \qquad (3.1.26)$$

$$\frac{1}{1 - L_f C_f \omega^2} = 0.03 \qquad (3.1.27)$$

$$\left| \frac{1}{f^2 \frac{X_L}{X_C} - 1} \right| \leq 0.03 \qquad (3.1.28)$$

$$\frac{X_L}{X_C} \geq \frac{34.2}{f^2} \qquad (3.1.29)$$

Where, f=corner or cutoff frequency

So, from this we can find out the L and C for the filter.

3.2.Determining Parasitics For Inverter Design

A parasitic element is a undesirable circuit element (resistance, inductance or capacitance) in a electrical circuit. This parasitic element changes the designed value of the circuit element due to which the device efficiency gets reduce and also it hinders the energy flow leading to the undesirable energy losses. So, while designing any circuit the parasitics must be considered and the circuit must be made in a way such that the value of parasitic elements must be minimal. Here is the method for determining the parasitics in VSI.

3.2.1 Mathematical Modelling

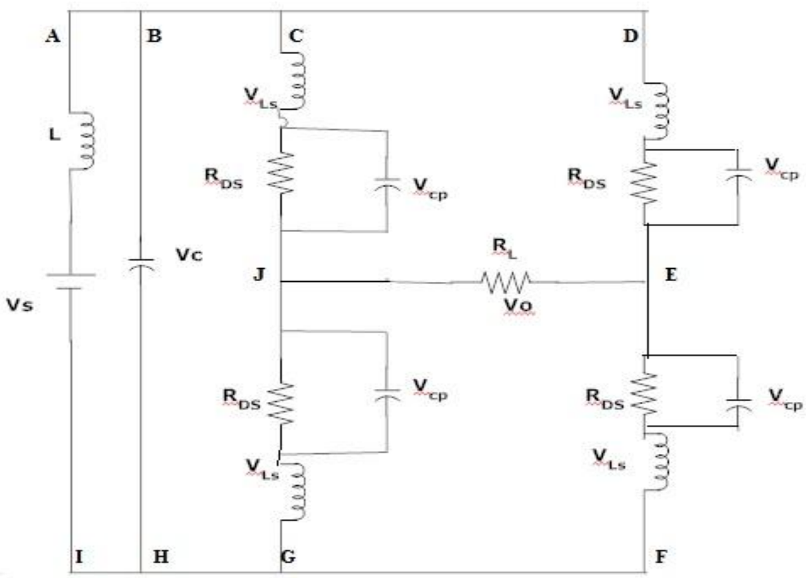


Fig16:VSI with parasitics

This is the proposed model of single-phase full–bridge inverter with parasitic elements. The analysis is divided into two parts; one is for turn-on state and the other is for turn-off state.

L= Parasitic inductance between source and input capacitance

Ls= Stray inductance of the wiring

R_{DS}=Drain-Source resistance

Cp=Parasitic capacitance

Applying KVL to ABHI

$$V_S = v_L + v_c \tag{3.2.1}$$

Applying KVL to BCJEFG

$$v_c = vo + 2 * v_{Cp} + i_{Ls} * R_L \tag{3.2.2}$$

$$v_c = 2Ls\frac{di_{Ls}}{dt} + 2v_{cp} + i_{Ls} * R_L \tag{3.2.3}$$

Applying KCL

$$i_{Ls} = i_{R_{DS}} + i_{Cp} \tag{3.2.4}$$

$$i = i_c + i_{Ls} \tag{3.2.5}$$

Since,

$$i_{R_{DS}} = \frac{v_{Cp}}{R_{DS}} \tag{3.2.6}$$

$$i_{Cp} = Cp\frac{dv_{Cp}}{dt} \tag{3.2.7}$$

$$i_{Ls} = \frac{v_{Cp}}{R_{DS}} + Cp\frac{dv_{Cp}}{dt} \tag{3.2.8}$$

Assuming identical device parameters and parasites

A. Parasitic Capacitance Voltage

$$v_{Cp} = \frac{v_c}{2} - Ls\frac{di_{Ls}}{dt} - \frac{v_o}{2} \tag{3.2.9}$$

$$v_{Cp} = \frac{v_c}{2} - Ls\frac{d}{dt}\left[\frac{v_{Cp}}{R_{DS}} + Cp\frac{dv_{Cp}}{dt}\right] - \frac{v_o}{2} \tag{3.2.10}$$

$$\frac{v_c - v_o}{2L_sC_p} = \frac{d^2v_{Cp}}{d^2t} + \frac{dv_{Cp}}{R_{DS}C_pdt} + \frac{v_{Cp}}{L_sC_p} \tag{3.2.11}$$

CASE I: Natural Response

$$A^2 + \frac{A}{R_{DS}C_p} + \frac{1}{L_sC_p} = 0 \tag{3.2.12}$$

$$A^1, A^2 = -\frac{1}{2R_{DS}C_p} \pm \sqrt{(\frac{1}{2R_{DS}C_p})2 - \frac{1}{L_sC_p}} \tag{3.2.13}$$

CASE II: Forced Response

$$\frac{d^2k}{dt^2} + \frac{1}{R_{DS}C_p}\frac{dk}{dt} + \frac{k}{LsCp} = \frac{v_c - v_o}{2L_sC_p} \tag{3.2.14}$$

Hence, the result of forced equation is

$$k = \frac{v_C - v_O}{2R_{DS}} \tag{3.2.15}$$

B. Parasitic Inductive Current

$$\frac{v_c - v_o}{2R_{DS}L_sC_p} = \frac{d^2i_{Ls}}{dt^2} + \frac{di_{Ls}}{R_{DS}C_pdt} + \frac{i_{Ls}}{L_sC_p} \tag{3.2.16}$$

CASE I: Natural Response

$$A^2 + \frac{A}{R_{DS}C_p} + \frac{1}{L_sC_p} = 0 \tag{3.2.17}$$

$$A^1, A^2 = -\frac{1}{2R_{DS}C_p} \pm \sqrt{(\frac{1}{2R_{DS}C_p})2 - \frac{1}{L_sC_p}} \tag{3.2.18}$$

CASE II: Forced Response

$$\frac{d^2k}{dt^2}+\frac{1}{R_{DS}C_p}\frac{dk}{dt}+\frac{k}{LsCp}=\frac{v_c-v_o}{2L_sC_p} \quad (3.2.19)$$

The result of force response is

$$\frac{v_c-v_o}{2R_{DS}}=\text{k} \quad (3.2.20)$$

CASE III: Complete Response

Mode 1: T1 and T2 (Turn ON)

$$i_{LS}(t)=i_{LN}+i_{LF} \quad (3.2.21)$$

$$i_{Ls}(t)=e^{\alpha t}(Be^{\beta t}+Ce^{-\beta t})+\frac{v_c-v_o}{2R_{DS}} \quad (3.2.22)$$

$$\alpha=-\frac{1}{2R_{DS}C_p} \quad (3.2.23)$$

$$\beta=\sqrt{(\frac{1}{2R_{DS}C_p})^2-\frac{1}{L_sC_p}} \quad (3.2.24)$$

$$v_{Cp}(t)=i_{Cp}(t)*R_{DS} \quad (3.2.25)$$

Mode2: T1 and T2 (Turn OFF)

$$v_{Cp}(t)=v_{CN}+v_{CF} \quad (3.2.26)$$

$$v_{Cp}(t)=e^{\alpha t}(Y\cos\beta t+Z\sin\beta t)+\frac{v_C-v_O}{2} \quad (3.2.27)$$

Through these equations we can determine the parasitic inductance and parasitic capacitance. So, the designed should be proposed in such a way to minimise the parasitics effects to maintain the efficiency of the inverter and smooth energy flow in the circuits.

3.3. PI Control

Proportional Integral (PI) control in VSI provides superior control over traditional Pulse Width Modulation or Sinusoidal Pulse Width Modulation (SPWM). In order to obtain a smooth desirable waveform at the output side, the switching frequency must be constant and should be independent of output frequency and this can be achieved by PI Control.

Advantages of PI Control:

- Fixed inverter switching frequency resulting in known harmonics.
- Instantaneous control and wave shaping.

3.3.1. PI Control Structure:

When a load is connected to the inverter output. The output voltage at the load side is sensed by means sensors and it is feedback to a comparator or subtractor which compares this load output with the reference signal (desired signal) and it produces the voltage error signal. This instantaneous error is fed to a proportional-integral (PI) controller. The integral term in the PI controller improves the tracking by reducing the instantaneous error between the reference and the actual voltage. .The error is forced to remain within the range defined by the amplitude of the triangular waveform. The resulting error signal is compared with a triangular carrier signal and intersections decide the switching frequency and pulse width.

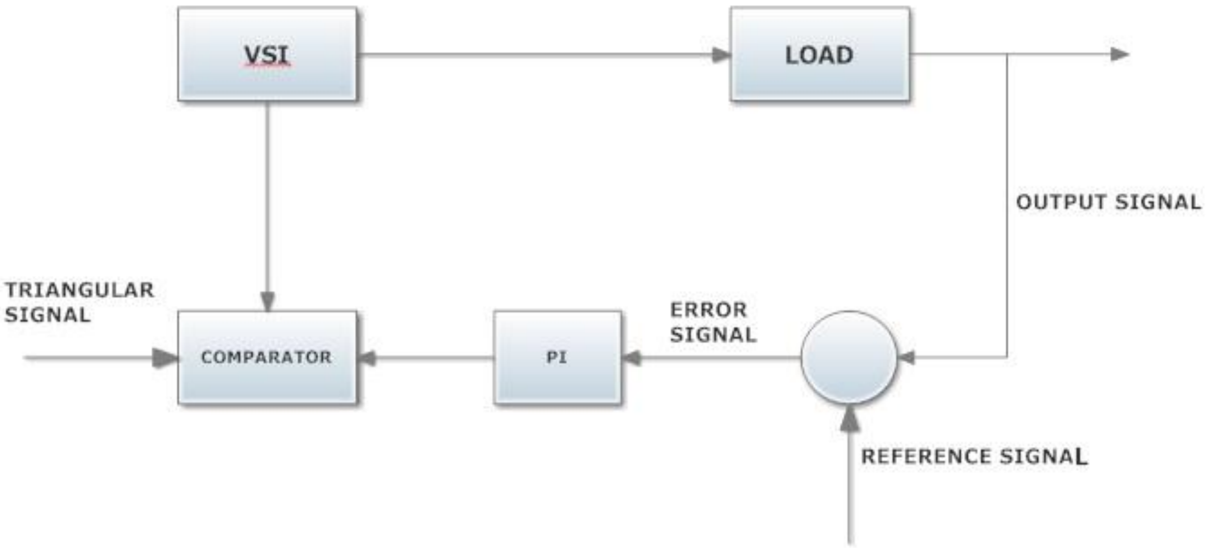


Fig17: VSI PI Control

PI controller is a feedback controller which detects the error value which is the difference of the output signal and the desired or reference signal. PI controller works to minimise this error by controlling the system inputs. PI controller has two elements namely Proportional (P) and Integral (I). Proportional part reduces the error while Integral part reduces the offset. P depends on present error and I depends on past errors. So, step response of a system can be improved by using PI controller.

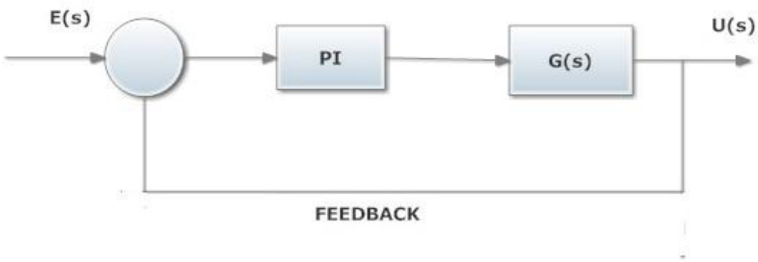


Fig 18: PI Control

After installing a PI Controller block the new response of the system will be

$$\frac{U(s)}{E(s)} = PI * G(s)$$

$$PI = Kp + \frac{Ki}{s}$$

$$\frac{U(s)}{E(s)} = (Kp + \frac{Ki}{s}) * G(s)$$

Now PI element gains, Kp(proportional gain) and Ki(integral gain) should be tuned to obtain a better system response. The effect of each parameters value on increasing is given below.

Response	Rise Time	Overshoot	Settling Time	Steady State Error
k_p	Decrease	Increase	Minor change	Decrease
k_i	Decrease	Increase	Increase	Eliminate

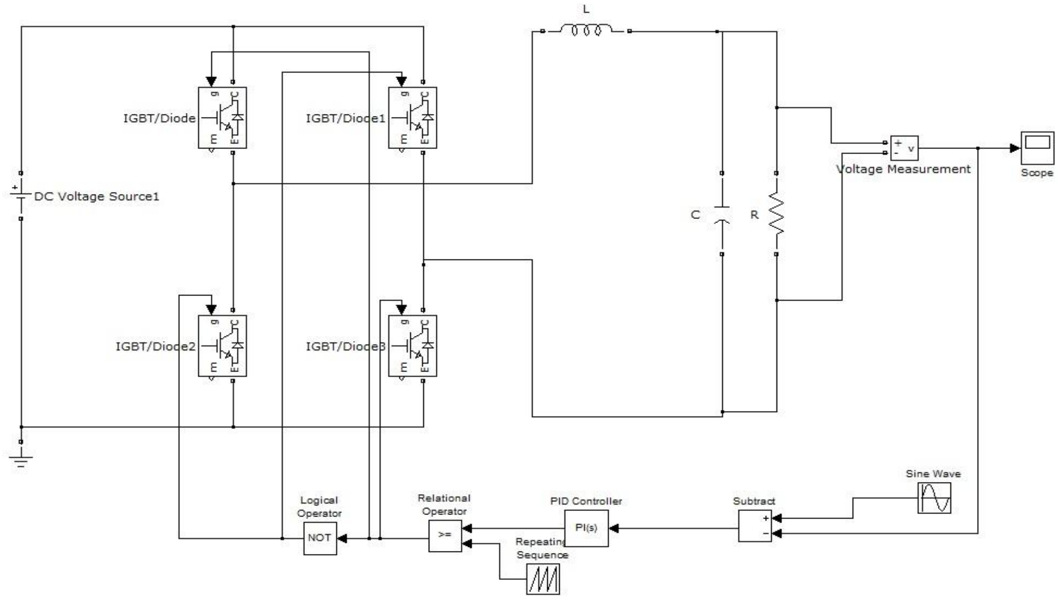
Table18: PI Tuning

CHAPTER IV

Results and Conclusion

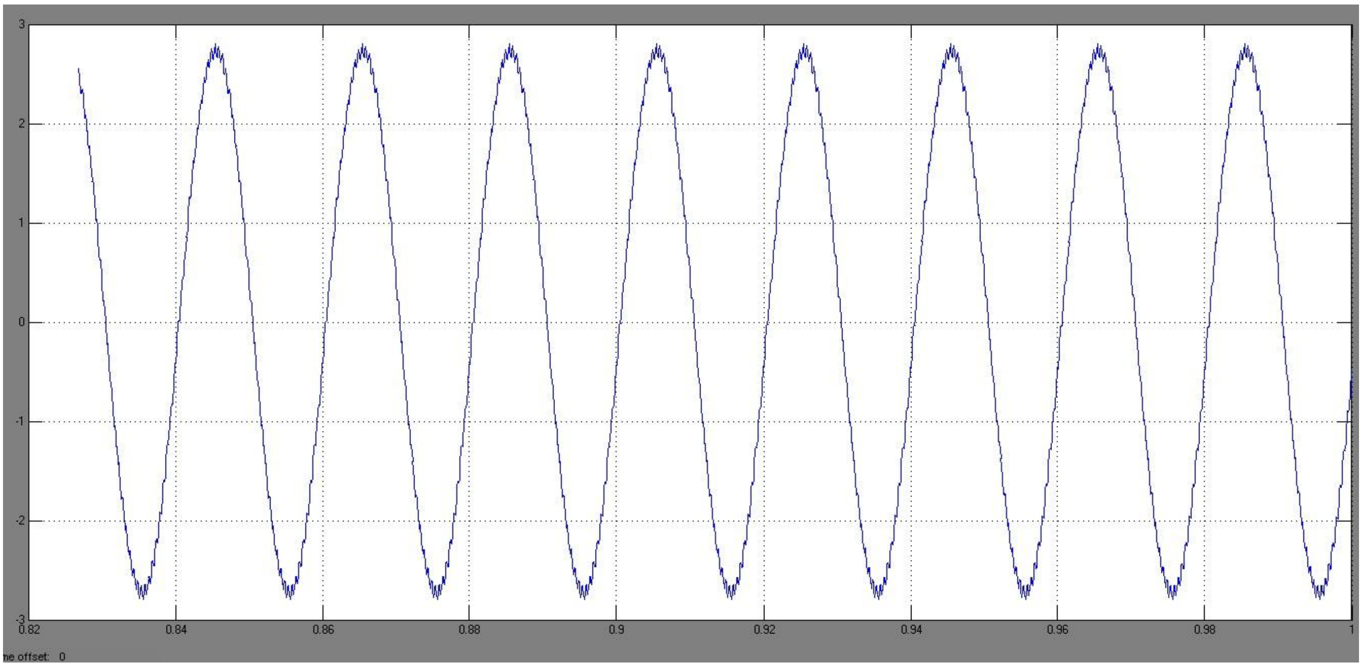
4. 1. Simulation Results

4.1. 1. Simple Inverter (Simulink Model)



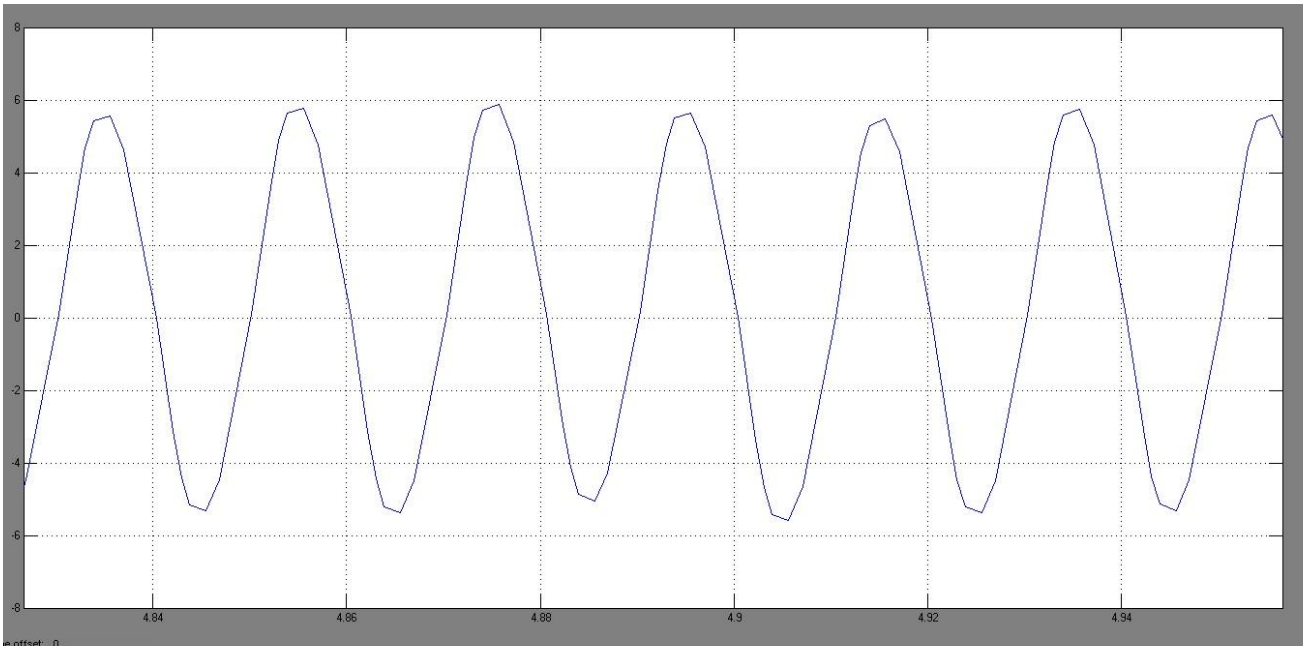
(L= 100mH,C= 1000µF)

Voltage Waveform



The voltage waveform obtained is nearly sinusoidal with little bit of distortions.

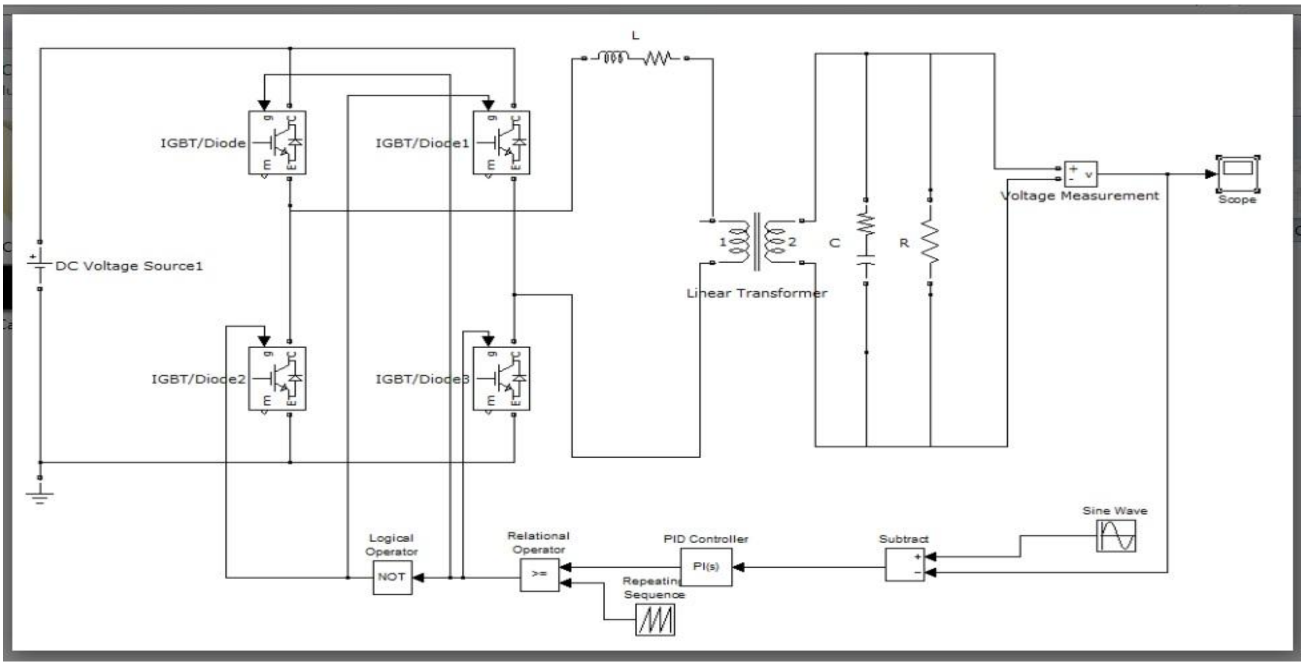
Current Waveform



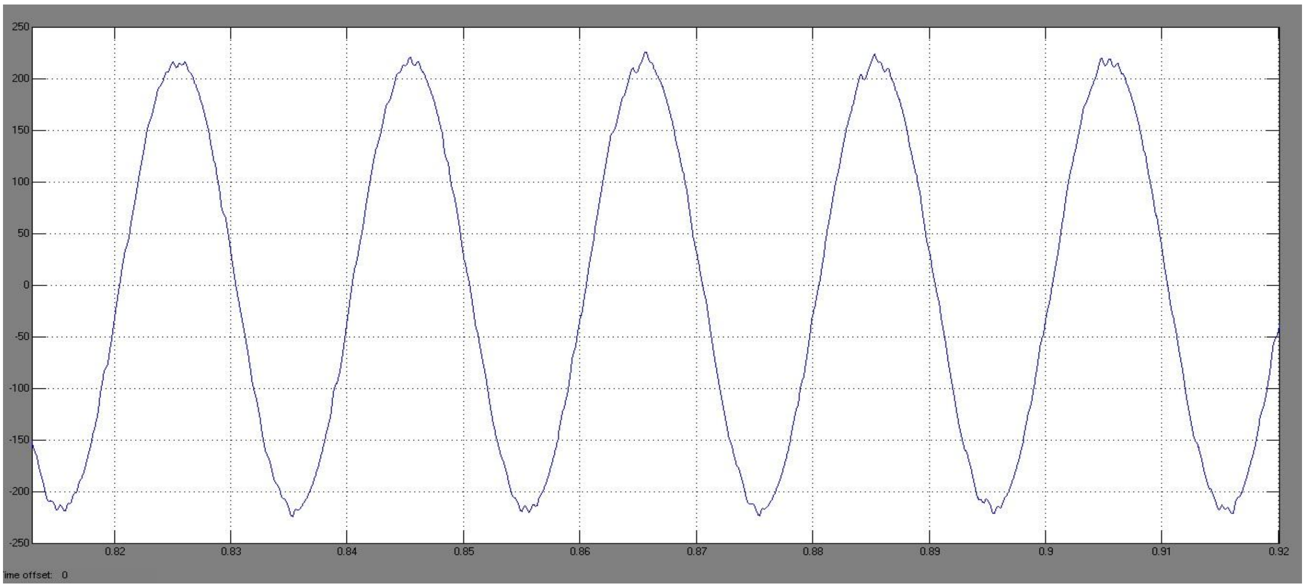
4.1.2. Practical Inverter (Simulink Model)

(v_{in}=24V DC, V_{out}=220V AC)

(R_L=1mΩ, L=152mH,Rc=10Ω,C=3200µF,R=1800Ω)

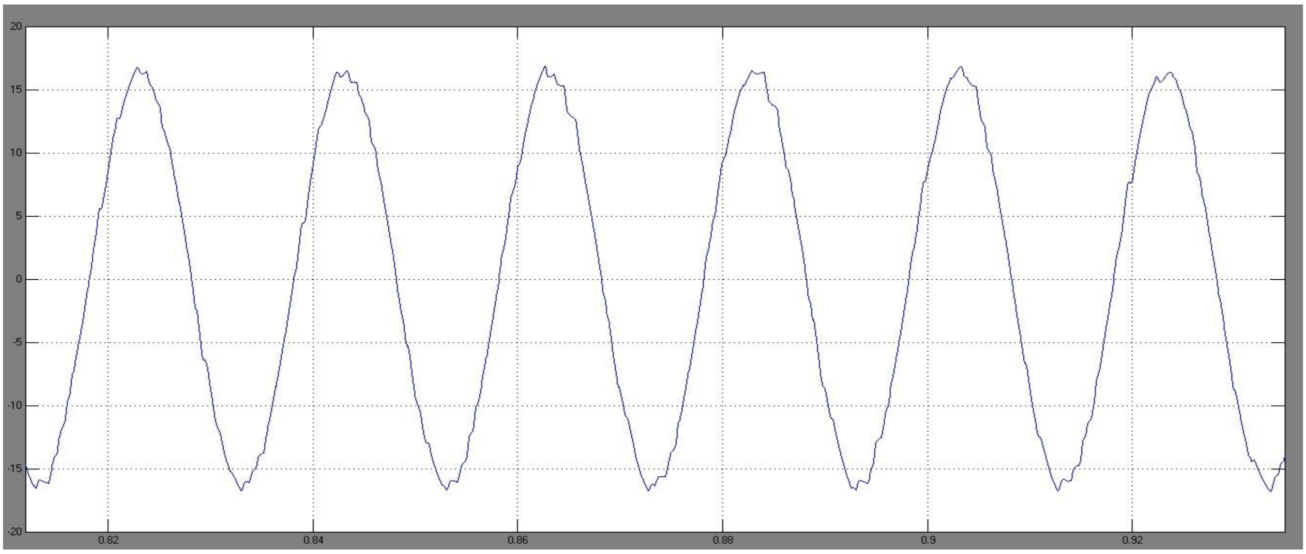


Voltage Waveform



Inverter input is a 24V dc and the output is 220V AC. This inverter can be used for household appliances.

Current Waveform



The current value depends on load and its waveform is also sinusoidal.

4.2. Conclusion

This Thesis Report deals with the analysis of Single Phase Sinusoidal Pulse Width Modulation (SPWM)-VSI. It includes both simple and practical SPWM-VSI. The Simulink model for both simple and practical inverter has been simulated in MATLAB. Its various parameters such as L and C for LC Filter design, k_p and k_i for PI controller and parasitics has been calculated for Simulink modelling and then simulated. These parameters are varied and the resulting voltage and current graphs has been studied.

6. Future Work

The future work includes improving the stability of the system and also to study various instability in SPWM-VSI with harmonic analysis and ways to eliminate it and to design an actual household SPWM-VSI with a better controller design.

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